Synchronization for 802.11a
Outline

- Timing synchronization
  - Frame detection

- Frequency synchronization
  - Carrier frequency offset (CFO) synchronization
  - Math analysis of inter carrier interference (ICI)
  - SNR degradation between OFDM and single carrier systems
Frame detection is the task of finding an approximate estimate of the start of the preamble of an incoming packet.

It is the first data-processing block of 802.11a baseband receiver.

As such it is the first synchronization algorithm that is performed, so the rest of the synchronization process is dependent on good packet detection performance.
Structure of 802.11a is follows as:

- The parts from $t_1$ to $t_{10}$ are short training symbols, that are all identical and 16-samples.
- The structure of the WLAN preamble enables the receiver to use a very simple and efficient algorithm to do *frame detection*, and *frequency synchronization*. 
timming synchronization (I)

Using correlator:

\[ r_n \xrightarrow{\times} C \xrightarrow{|.|^2} \frac{\sum 1_{16,16} Z^{-D} C}{p_n} \xrightarrow{(.)^2} m_n \]

From the delay correlate structure, the decision is calculate as

\[ c_n = \sum_{k=0}^{L-1} r_{n+k} r_{n+k}^* \]
\[ p_n = \sum_{k=0}^{L-1} r_{n+k+D} r_{n+k+D}^* = \sum_{k=0}^{L-1} |r_{n+k+D}|^2 \]

\[ m_n = \frac{|c_n|^2}{(p_n)^2} \]

where \( D = 16, L = 16 \)
The double sliding window packet detection algorithm calculates two consecutive sliding windows of the received energy.

The basic principle is to form the decision variable $m_n$ as a ratio of total energy contained inside the two windows as follows:
Cont'd

- When the packet edge starts to cover the A window, the energy in the A window gets higher until the point where A is totally contained inside the start of the packet.
- The packet detection is declared when $m_n$ crosses over the threshold value $Th$.
- The algorithm described as

$$a_n = \sum_{m=0}^{M-1} r_{n-m}^* r_{n-m} = \sum_{m=0}^{M-1} |r_{n-m}|^2$$

where $M$ is length of window A.

$$b_n = \sum_{l=1}^{L} r_{n+l}^* r_{n+l} = \sum_{l=1}^{L} |r_{n+l}|^2$$

where $L$ is length of window B.

$$m_n = \frac{a_n}{b_n}$$
Carrier frequency synchronization for 802.11a

Wei-Wen Hu (胡偉文)
The effect of CFO

- In OFDM system, if there is any mismatch between the frequency and phase of Tx and Rx, it will result CFO.
- There are two destructive effects caused by CFO:
  - One is the reduction of signal amplitude.
  - It will result in ICI which is caused by the loss of the subcarriers orthogonality.
- The FFT output for each subcarrier will contain interference term from other subcarrier.
Math analysis of ICI

- An OFDM transmission symbol is given by the $N$ point complex modulation sequence

$$x_n = \frac{1}{N} \sum_{k=-k}^{k} X_k e^{\frac{j2\pi nk}{N}}$$

- After passing through channel, the received sequence can be expressed as

$$y_n = \frac{1}{N} \left[ \sum_{k=-k}^{k} X_k H_k e^{\frac{j2\pi n(k+\varepsilon)}{N}} \right] + w_n$$

- The output of the FFT for $k$th subcarrier consisting of three components

$$Y_k = \sum_{n=0}^{N-1} y_n e^{-\frac{j2\pi kn}{N}} = S_k + I_k + W_k$$
Then, the variance of interference signal

\[ I_k = \sum_{l=-k}^{k} (X_l H_l) \left\{ \frac{\sin \pi \varepsilon}{N \sin \left( \frac{\pi (l-k+\varepsilon)}{N} \right)} \right\} e^{\frac{j \pi (N-1)}{N}} e^{-\frac{j \pi (l-k)}{N}} \]

Therefore, the variance of interference signal is

\[ E\left( I_k^2 \right) \leq 0.5947 |X|^2 |H|^2 (\sin \pi \varepsilon)^2 \]

Generally, the interference power is proportional to the frequency offset.
The degradation $D$ is given by

$$D \approx \frac{10}{\ln 10} \frac{1}{3} \left( \pi N \frac{\Delta f}{R} \right)^2 \frac{E_s}{N_0} \quad \text{OFDM}$$

$$D \approx \frac{10}{\ln 10} \frac{1}{3} \left( \pi \frac{\Delta f}{R} \right)^2 \quad \text{Single carrier}$$

where $R = N / T$ for OFDM, $R = 1 / T$ for single carrier.
CFO estimation

- Using the correlator that takes maximum likelihood estimation (MLE) to estimate the CFO

- The received signal is

\[ r_n = s_n e^{j2\pi f_{tx} n T_s} e^{-j2\pi f_{rx} n T_s} \]

\[ = s_n e^{j2\pi (f_{tx} - f_{rx}) n T_s} \]

\[ = s_n e^{j2\pi f_\Delta n T_s} \]

- The correlator output is

\[ z = \sum_{k=0}^{L} r_k r_k^* \]

\[ = e^{-j2\pi f_\Delta DT_s} \sum_{k=0}^{L} |s_n|^2 \]
Finally, the frequency error estimator is formed as

\[ \hat{f}_\Delta = -\frac{1}{2\pi DT_s} \arg(z) \]

The algorithm is simple and can use the same hardware of the delay and correlate algorithm.
Algorithm of CFO

The CFO algorithm is based on packet detection algorithm when packet is detected over the threshold.

The algorithm is described as

\[
M(n) = \frac{C(n)}{P(n)} = \frac{\sum_{k=0}^{L-1} r_{n+k}^* r_{n+k+D}^*}{\sum_{k=0}^{L-1} |r_{n+k+D}|^2}
\]

Then, the coarse CFO is

\[
\Delta f_{coarse} = \frac{1}{2\pi DT_s} \arg \left( C(n) \right) \mid_{M(n) > TH}
\]
Algorithm of the fine CFO

During short preamble, we get the coarse CFO, in this algorithm the correlator can be used again.

The algorithm is described as

\[ r'_\text{long} (k) = r_\text{long} (k) \exp \left( -j 2\pi k \Delta \hat{f}_\text{coarse} \right) \]
\[ = r_\text{long} (k) \exp \left( -jk \cdot \arg \left( C(m) \right) / DT_s \right) \]

The fine estimation of CFO is

\[ \Delta \hat{f}_\text{fine} = \frac{1}{2\pi N_L T_s} \arg \left( \sum_{l=N_L}^{2N_L-1} r'_l \left( \frac{r'_{l-N_L}}{\cdot} \right)^* \right) \]
\[ N_L = 64 \]
After finishing the acquisition of CFO, both coarse and fine estimation is available.

Therefore, the received signal is described as

\[ \hat{r}_k = r_k \exp \left( -j 2\pi \left( \Delta \hat{f}_{\text{coarse}} + \Delta \hat{f}_{\text{fine}} \right) k \right) \]
Acquisition range

- In 802.11a standard, the maximum carrier frequency offset is
  \[ 5GHz \cdot 40 \text{ ppm} = 200KHz \]

- Finally, the frequency error estimator is formed as
  \[ f_\Delta = -\frac{1}{2\pi DT_s} \arg(z) \]

- For short preamble (D=16), thus the maximum frequency range is
  \[ f_{\Delta_{\text{max}}} = \frac{1}{2 \times 16 \times 50 \times 10^{-9}} = 625 \text{ kHz} \]

- For long preamble (D=64), thus the maximum frequency range is
  \[ f_{\Delta_{\text{max}}} = \frac{1}{2 \times 64 \times 50 \times 10^{-9}} = 156.25 \text{ kHz} \]
Other method (using cyclic prefix--MLE)

